Math 60 9.2 n^{th} roots

* Note *: Rational exponents will be done after 9.6

Objectives:

1) Evaluate n^{th} radicals
2) Simplify $(\sqrt[n]{a})^n$
3) Simplify $\sqrt[n]{a^n}$

Recall from 9.1: The square root is the reverse process of exp. 2.

perfect squares	square roots
$1^2 = 1$	$\sqrt{1} = \sqrt{1^2} = 1$
$2^2 = 4$	$\sqrt{4} = \sqrt{2^2} = 2$
$3^2 = 9$	$\sqrt{9} = \sqrt{3^2} = 3$
$4^2 = 16$	$\sqrt{16} = \sqrt{4^2} = 4$
$5^2 = 25$	$\sqrt{25} = \sqrt{5^2} = 5$
$6^2 = 36$	$\sqrt{36} = \sqrt{6^2} = 6$
$7^2 = 49$	$\sqrt{49} = \sqrt{7^2} = 7$
$8^2 = 64$	$\sqrt{64} = \sqrt{8^2} = 8$
$9^2 = 81$	$\sqrt{81} = \sqrt{9^2} = 9$
$10^2 = 100$	$\sqrt{100} = \sqrt{10^2} = 10$

In 9.2 will will reverse the process for exponents 3, 4, and 5.

perfect cubes	cube roots
$1^3 = 1$ *	$\sqrt[3]{1} = \sqrt[3]{1^3} = 1$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
$4^3 = 64$ *	$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$
$5^3 = 125$	$\sqrt[3]{125} = \sqrt[3]{5^3} = 5$
$6^3 = 216$	$\sqrt[3]{216} = \sqrt[3]{6^3} = 6$
$7^3 = 343$	$\sqrt[3]{343} = \sqrt[3]{7^3} = 7$
$8^3 = 512$	$\sqrt[3]{512} = \sqrt[3]{8^3} = 8$
$9^3 = 729$	$\sqrt[3]{729} = \sqrt[3]{9^3} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = \sqrt[3]{10^3} = 10$

Note: 1 and 64 are also perfect squares!

Math 60 9.2

Vocabulary and notation

$\sqrt{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, $\sqrt[5]{\ }$ .... $\sqrt[n]{\ }$ are called <u>radicals</u>.

The $\sqrt{\ }$ part of any radical is called the <u>radical</u>.

$\sqrt[3]{\ }$ $\sqrt[4]{\ }$ $\sqrt[5]{\ }$ ... $\sqrt[n]{\ }$

The small number is called the <u>index</u>.
The index tells us what exponent we are un-doing.

$\sqrt{\ }$ square roots have index 2, but we imply it rather than writing it.

The number on the inside of the radical is called the <u>argument</u> or the <u>radicand</u>.

("Argument" is a more general word, referring to the number put in to any function, while radicand is specific to radicals.)

① $\sqrt[3]{8}$ is spoken "3rd root of 8" index 3 radicand 8

② $\sqrt[4]{81}$ is spoken "4th root of 81" index 4 radicand 81

③ $\sqrt[n]{x}$ is spoken "n^{th} root of x" index n radicand x.

Remember that negative numbers and square roots are problematic:

⑦ $\sqrt{-4}$ = not a real number (9.9)

⑧ $\sqrt{\text{anything}} \neq$ negative because $\sqrt{(+)} = (+)$
$\sqrt{(0)} = 0$
$\sqrt{(-)}$ = imaginary not real.

This is because $2^2 =$ pos $(-2)^2 =$ pos $(\text{nothing})^2 =$ neg

Math 60 9.2

But notice when we have perfect cubes

$(-1)^3 = -1$ so $\sqrt[3]{-1} = \sqrt[3]{(-1)^3} = -1$

$(-2)^3 = -8$ so $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

$(-3)^3 = -27$ so $\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$

$(-4)^3 = -64$ so $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$

$(-5)^3 = -125$ so $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

$(-6)^3 = -216$ so $\sqrt[3]{-216} = \sqrt[3]{(-6)^3} = -6$

$(-7)^3 = -343$ so $\sqrt[3]{-343} = \sqrt[3]{(-7)^3} = -7$

$(-8)^3 = -512$ so $\sqrt[3]{-512} = \sqrt[3]{(-8)^3} = -8$

$(-9)^3 = -729$ so $\sqrt[3]{-729} = -9$

$(-10)^3 = -1000$ so $\sqrt[3]{-1000} = -10.$

With cube roots, negative numbers are permitted in both places!

$\sqrt[3]{\text{negative}} = \text{negative}$

negative # in (radicand)	=	negative # out (answer)

Math 60 9.2

Practice: Evaluate exactly or use a calculator and round result to the nearest hundredth.

(9) $\sqrt[3]{27}$

$3^3 = 27$

so $\sqrt[3]{27} = \boxed{3}$

(10) $\sqrt[3]{-125}$

$(-5)^3 = -125$

So $\sqrt[3]{-125} = \boxed{-5}$

(11) $\sqrt[3]{17}$

17 is not a perfect cube

calculator gives 2.571281591

rounded $\boxed{2.57}$

If your calculator does not have a $\sqrt{\ }$ or $\sqrt[3]{\ }$ or $\sqrt[n]{\ }$ button, you can use your exponent key

(12) $\sqrt{4} = 4^{1/2} = 4^{.5} = 2$

(13) $\sqrt[3]{8} = 8^{1/3} = 8^{\text{don't use decimal}} = 8\text{^}(1/3) = 2$

(14) $\sqrt[4]{16} = 16^{1/4} = 16^{.25} = 2$

$$\boxed{\begin{array}{c}\text{Fraction exponents mean radicals} \\ x^{1/n} = \sqrt[n]{x}\end{array}}$$

* We will do this later

Math 60 9.2

For fourth powers, we have

$1^4 = 1^*$ | $\sqrt[4]{1} = 1$
$2^4 = 16^*$ | $\sqrt[4]{16} = 2$
$3^4 = 81^*$ | $\sqrt[4]{81} = 3$
$4^4 = 256^*$ | $\sqrt[4]{256} = 4$
$5^4 = 625^*$ | $\sqrt[4]{625} = 5$
$6^4 = 1296^*$ | $\sqrt[4]{1296} = 6$

Every 4th power is also a square

BUT

$(-1)^4 = +1$
$(-2)^4 = +16$

so

$\sqrt[4]{-1}$ is not a real number
$\sqrt[4]{-16}$ is not a real number

$\boxed{\sqrt[4]{\text{any neg}} \text{ is not a real number.}}$

For fifth powers, we have:

$1^5 = 1$ | $\sqrt[5]{1} = 1$
$2^5 = 32$ | $\sqrt[5]{32} = 2$
$3^5 = 243$ | $\sqrt[5]{243} = 3$
$4^5 = 1024$ | $\sqrt[5]{1024} = 4$

and

$(-1)^5 = -1$ | $\sqrt[5]{-1} = -1$
$(-2)^5 = -32$ | $\sqrt[5]{-32} = -2$
$(-3)^5 = -243$ | $\sqrt[5]{-243} = -3$
$(-4)^5 = -1024$ | $\sqrt[5]{-1024} = -4$

And again negative values are valid and permitted in both places

$\boxed{\sqrt[5]{\text{neg}} = \text{neg}}$

What can you surmise about:

$\sqrt[6]{\text{negative \#}}$

$\sqrt[7]{\text{negative \#}}$

$\sqrt[n]{\text{negative \#}}$?

Math 60 9.2

If the index n is <u>even</u> :	$\sqrt[n]{neg}$	=	not a real #
If the index n is <u>odd</u> :	$\sqrt[n]{neg}$	=	negative #.

Practice continued:

(15) $\sqrt[4]{256}$

$256 = 4^4$

so $\sqrt[4]{256} = \boxed{4}$

(16) $\sqrt[4]{-16}$

$\left(\begin{smallmatrix}\text{no}\\ \#\end{smallmatrix}\right)^4 = \text{neg}$

so $\boxed{\text{not a real \#}}$

(17) $\sqrt[4]{53}$

53 is not a perfect fourth power

so calculator gives 2.698167876

$\boxed{2.70}$

(18) $\sqrt[3]{(-2)^3}$

Evaluate grouping symbols from inside out

$= \sqrt[3]{-8}$

3 index is odd, $(-2)^3 = -8$

$= \boxed{-2}$

Math 60 9.2

(19) $\sqrt[4]{(-2^4)}$

Evaluate grouping symbols from inside out

$= \sqrt[4]{16}$

$= \boxed{2}$ yes! The original # was negative 2, but the 4th root is +2.

(20) $\sqrt[5]{(-2)^5}$

$= \sqrt[5]{-32}$

$= \boxed{-2}$ index 5 is odd, negative values permitted.

(21) $\sqrt[6]{(-2)^6}$

$= \sqrt[6]{64}$

$= \boxed{2}$

In general:

$$\boxed{\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \quad (\text{can be negative}) \\ |a| & \text{if } n \text{ is even} \quad (\text{cannot be negative}) \end{cases}}$$

Simplify. Variables can be any real number.

(22) $\sqrt[5]{x^5} = \boxed{x}$ odd index, no absolute value x can be negative or positive result.

(23.) $\sqrt[4]{(n-3)^4} = \boxed{|n-3|}$ even index, must use absolute value

(24) $-\sqrt[6]{(-2)^6} = -\sqrt[6]{64} = \boxed{-2}$

⇑
neg outside is last part of calculation

Math 60 9.2

25) $\sqrt[3]{\frac{-27}{8}}$

$= \frac{\sqrt[3]{-27}}{\sqrt[3]{8}}$

Just as we could split apart square roots, we can split apart n^{th} roots.

$= \boxed{\frac{-3}{2}}$

A note about instructions:

If instructions say:

1) "Assume variables can be any real number."

This means both positive and negative values, and zero.

* Since negative #s are the troublemakers, we may need absolute values.

2) "Assume variables are positive."

This means only positive #s are allowed. Since negatives and zero are not permitted, no absolute values will be needed.

3) "Assume variables are non-negative."

This means positive #s and zero are allowed.

Since no negatives are permitted, no absolute values will be needed.

Math 60 9.2

Simplify. Assume variables can be any real number.

26 $\sqrt[3]{(x-1)^3} + \sqrt[5]{32}$

$= (x-1) + 2$

$= x - 1 + 2$

$= \boxed{x+1}$

odd index

$\sqrt[3]{neg} = neg$

so

$\sqrt[3]{x^3} = x$, no abs values.

27 $\dfrac{5\sqrt[7]{x^7}}{\sqrt[3]{x^3}}$

both 7 and 3 are odd indices

$\Rightarrow$ no absolute values.

$= \dfrac{5x}{x}$

$= \boxed{5}$

$\dfrac{x}{x} = 1$

28 $\sqrt[4]{(x-1)^4} + \sqrt[4]{16}$

$= \boxed{|x-1| + 2}$

4 = even index, must have abs. values.

cannot simplify further.

29 $\sqrt{x^2 - 8x + 16}$

$= \sqrt{(x-4)(x-4)}$ factor

$= \sqrt{(x-4)^2}$ write as exponent 2.

$= \boxed{|x-4|}$ even index 2, must have absolute values.

Review

30 $\sqrt[3]{-64}$

$= \boxed{-4}$

31 $\sqrt{64}$

$= \boxed{8}$

32 $-\sqrt[4]{256}$

$= -\sqrt[4]{2^8}$

$= -2^{8/4}$

$= -2^2$

$= \boxed{-4}$

33 $\sqrt[3]{\dfrac{-8}{125}}$

$= \dfrac{\sqrt[3]{-8}}{\sqrt[3]{125}}$

$= \boxed{\dfrac{-2}{5}}$